A Brief Introduction to Astrophysics

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What, Why & Why Me?

- What? Astrophysics!
- Why? Astronomy is a **quantitative science** and that means it is mathematical.
- Why Me? I am a student, *not an expert*, at astrophysics.

Question?

- ★ How can you possibly describe a star with math?

Overview

- Qualities, quantities, equalities and proportionalities
- Light
- Stars
- Galaxies
- Cosmology

Qualities vs. Quantities

- When we speak **qualitatively** we say things like "The sun is hot" or "the Andromeda Galaxy is a very long ways away."
- When we speak **quantitatively** we say things like "The surface of the sun is 5500 degrees" or "the Andromeda Galaxy is 2.9 million light years away"

We can <u>measure</u> quantities.

Equality and Proportionality

- An equation is a quantitative statement of equality because two different expressions.

$$t = \frac{x}{v}$$

t is time
x is distance
v is velocity

- A proportionality is a qualitative statement of dependence of one quantity on another.

$$t \propto \frac{1}{v}$$

Let There Be Light

Light is a wave.

λ

Lambda is the wavelength of light.

The bigger lambda is the longer the wavelength is and the redder the color is.

Let There Be Light

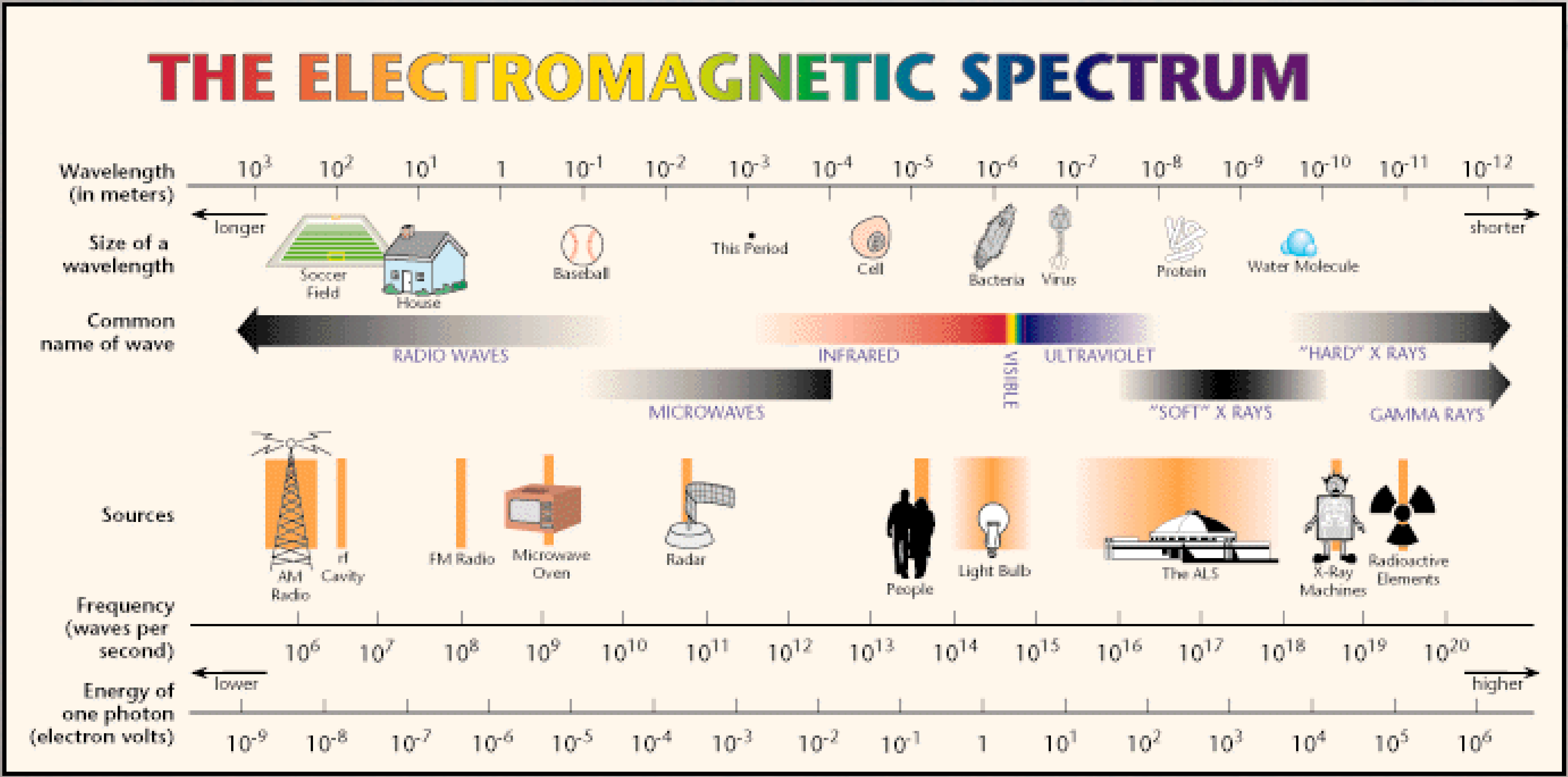
$$E = \frac{hc}{\lambda}$$

$$p = \frac{h}{\lambda}$$

Light is a particle called a photon.

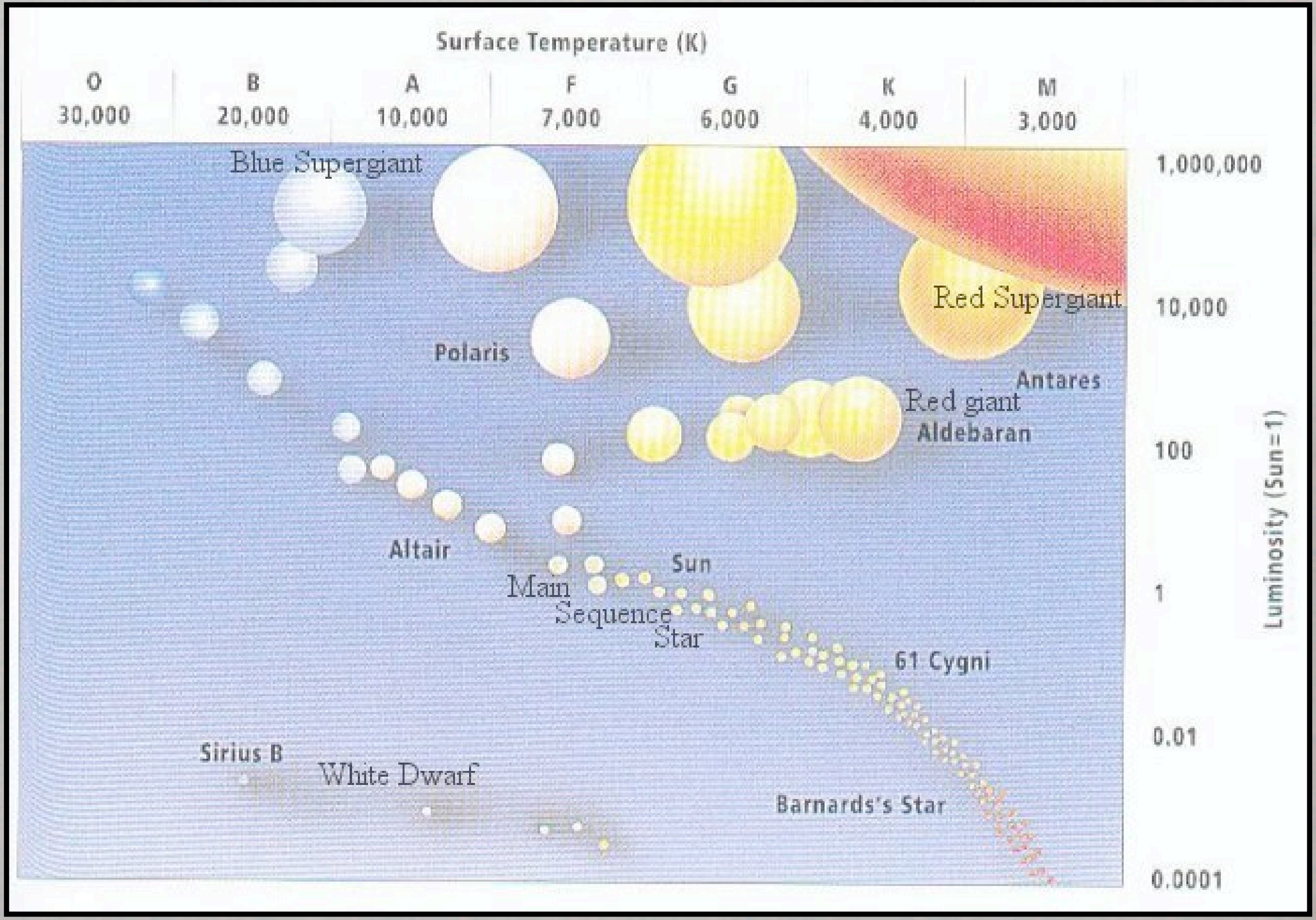
The energy of a photon is inversely proportional to the wavelength of the light.

The momentum of light is equal to Planck's constant divided by the wavelength of the light.


THE ELECTROMAGNETIC SPECTRUM
Wavelength (in meters)
10^3 10^2 10^1 1 10^-1 10^-2 10^-3 10^-4 10^-5 10^-6 10^-7 10^-8 10^-9 10^-10 10^-11 10^-12
longer
shorter
Size of a wavelength
Soccer Field
House
Baseball
This Period
Cell
Bacteria
Virus
Protein
Water Molecule
Common name of wave
RADIO WAVES
INFRARED
VISIBLE
ULTRAVIOLET
"HARD" X RAYS
MICROWAVES
"SOFT" X RAYS
GAMMA RAYS
Sources
AM Radio
rf Cavity
FM Radio
Microwave Oven
Radar
People
Light Bulb
The ALS
X-Ray Machines
Radioactive Elements
Frequency (waves per second)
10^6 10^7 10^8 10^9 10^10 10^11 10^12 10^13 10^14 10^15 10^16 10^17 10^18 10^19 10^20
lower
higher
Energy of one photon (electron volts)
10^-9 10^-8 10^-7 10^-6 10^-5 10^-4 10^-3 10^-2 10^-1 1 10^1 10^2 10^3 10^4 10^5 10^6

Stellar Properties

- Mass
- Radius
- Temperature
- Luminosity
- Color
- Distance



Star Formation: The Jeans Mass

$$M_J = \frac{4}{\sqrt{\rho}}\left(\frac{kT}{Gm}\right)^{3/2}$$

When a star forms from a gas cloud there is a minimum mass of material that must be available to overcome the thermal motion of the particles.

for example:

100,000 H_2 atoms at 50°K (-220°C)

M_J=76 solar masses

Stars

$$E = \sigma T^4$$

$$L = \left(4\pi R^2\right)E$$

$$F = \frac{L}{4\pi d^2}$$

The **Energy** is proportional to the 4th power of the temperature.

The **Luminosity** is proportional to the square of the radius and the Energy.

The **Flux** is inversely proportional to the square of the distance.

Stars

$$m_2 - m_1 = 2.5 \log(F_1 / F_2)$$

The difference in magnitude of two stars is proportional to the logarithm of the proportion of the fluxes.

Dust

$$T_g = T_* \sqrt{\frac{R_*}{2d}}$$

$$d = \left(\frac{T_*}{T_g}\right)^2 \frac{R_*}{2}$$

The temperature of a dust grain is related to the temperature and radius of the star it orbits and its distance from the star.

This equation can be solved for the distance as a function of the temperature of the dust grain.

Dust

$$d = \left(\frac{T_*}{273^\circ K}\right)^2 \frac{R_*}{2} = 0.51 A.U.$$

$$d = \left(\frac{T_*}{373^\circ K}\right)^2 \frac{R_*}{2} = 0.95 A.U.$$

Oops! The temperatures here should be reversed.

Water freezes at 273˚K.
Water boils at 373˚K.

Earth is 1.0 A.U. from the Sun.

Conclusion: Liquid water is not possible on earth.

Orbits

$$F = ma$$

$$a = \frac{v^2}{r}$$

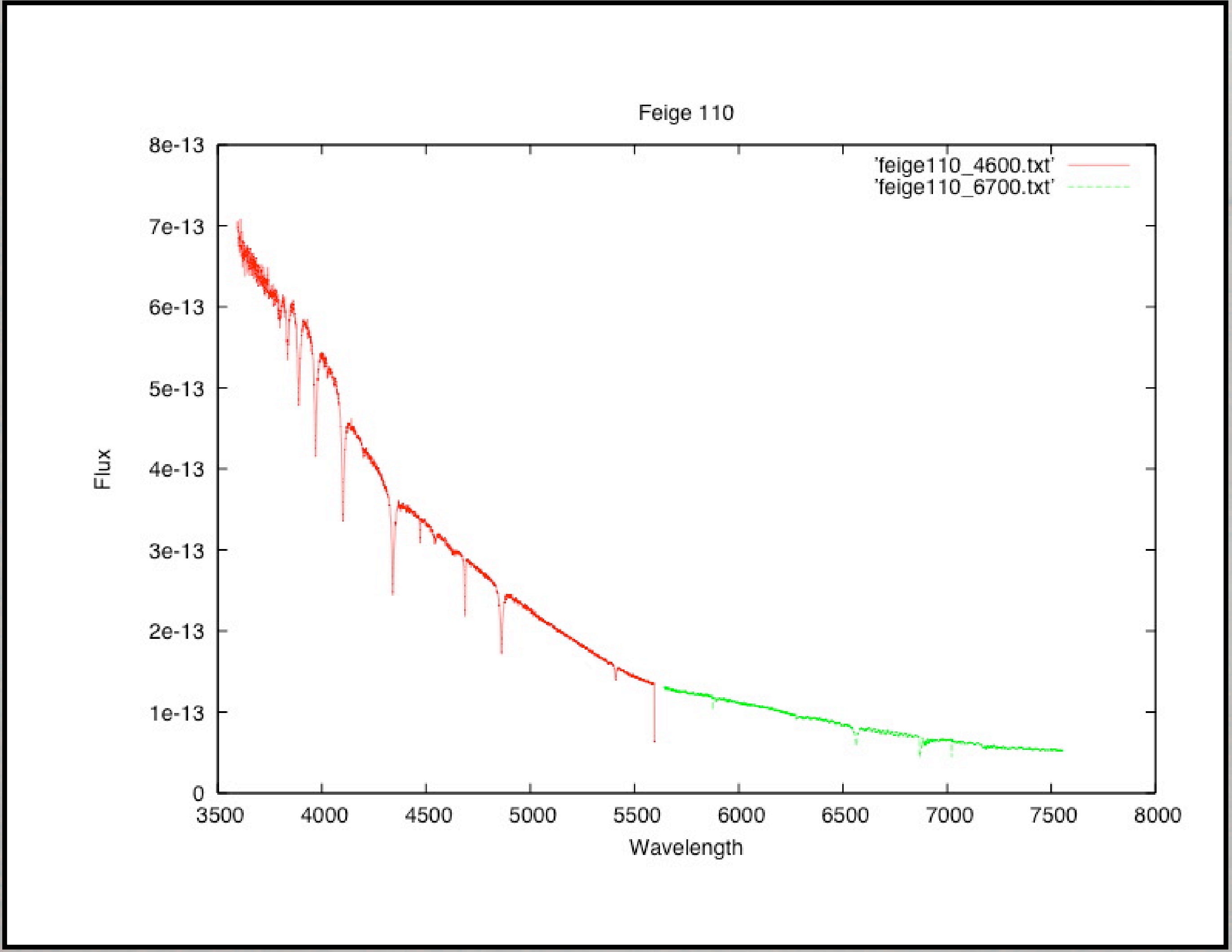
$$F = \frac{GMm}{r^2}$$

$$m\frac{v^2}{r} = \frac{GMm}{r^2}$$

$$M = \frac{v^2 r}{G}$$

Using first principles you can find an equation to calculate the mass of, say, the Sun or the Earth based on the speed and distance of an object orbiting it.

Spectroscopy



Stars have "fingerprints" which we can read in their spectrum.

Doppler Shift

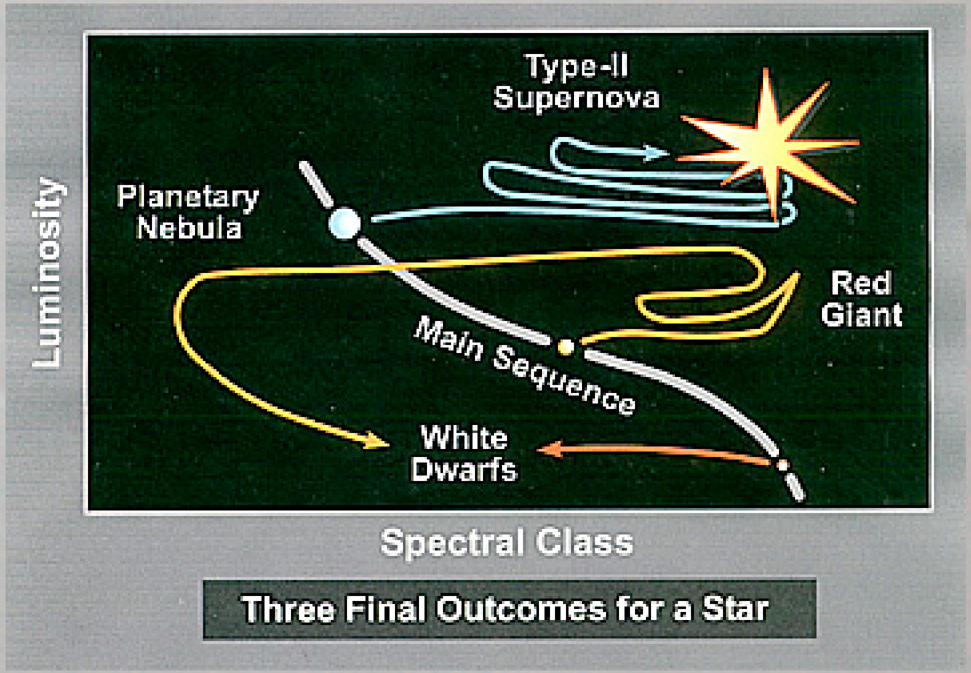
$$v_r = \left(\frac{\lambda}{\lambda_0} - 1 \right) c$$

$$v_r = \left(\frac{550.5}{550} - 1 \right) 3 \times 10^5 \, km/s = 273 km/s$$

You can measure the velocity of an object towards or away from us by measuring the doppler shift of the spectrum.

Stellar Old Age

- Low Mass Stars
  - Red Giants
  - Planetary Nebula
- High Mass Stars
  - White Dwarfs
  - Neutron Stars
  - Supernova
  - Black Holes



Black Holes

$$v_{esc} = \left(\frac{2GM}{R}\right)^{1/2}$$

$$R_S = \frac{2GM}{c^2}$$

The escape velocity is proportional to the Mass and inversely proportional to the Radius.

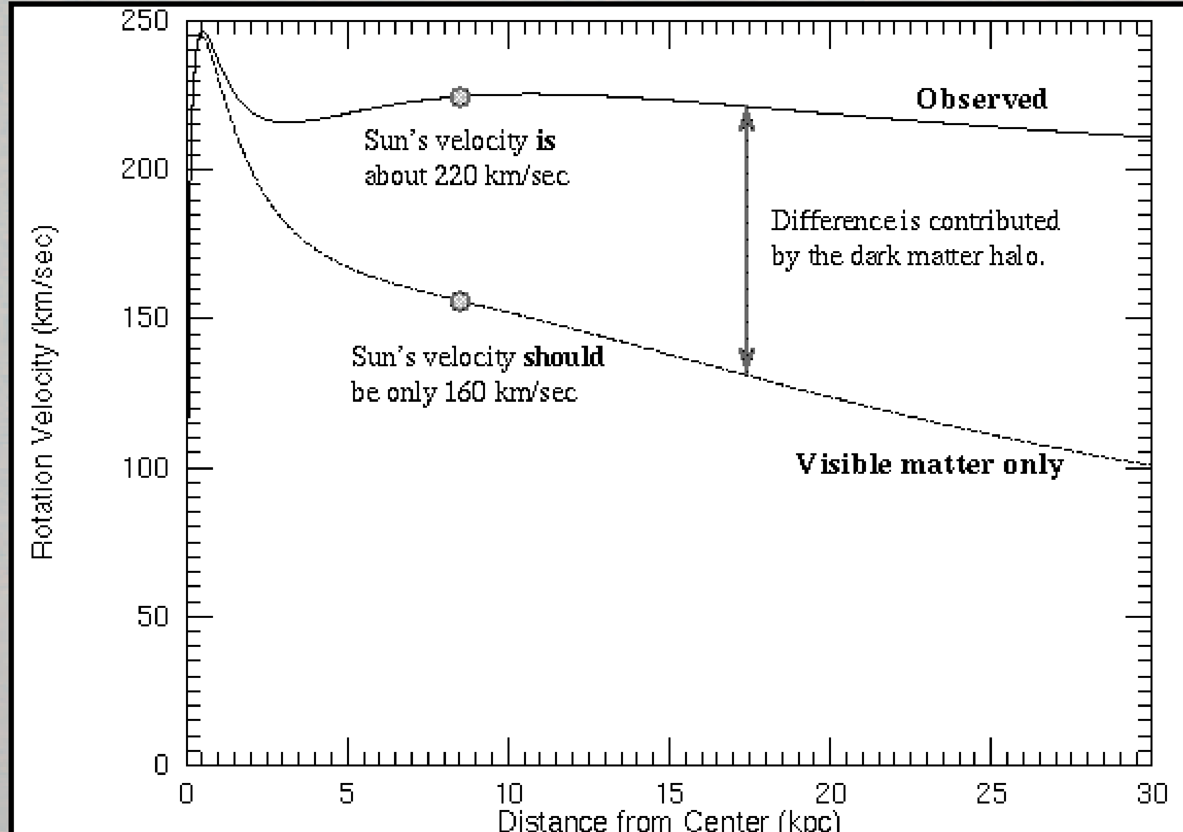
If you set *v* to the speed of light (c) and solve for the radius you get the radius at which a black hole forms.

Red Shift

$$z = \frac{\Delta\lambda}{\lambda}$$

The red shift, z, is a measure of distance based on recession velocity.

The Galaxy: Rotation



The gravity of the visible matter in the Galaxy is not enough to explain the high orbital speeds of stars in the Galaxy. For example, the Sun is moving about 60 km/sec too fast. The part of the rotation curve contributed by the visible matter only is the bottom curve. The discrepancy between the two curves is evidence for a **dark matter halo.**

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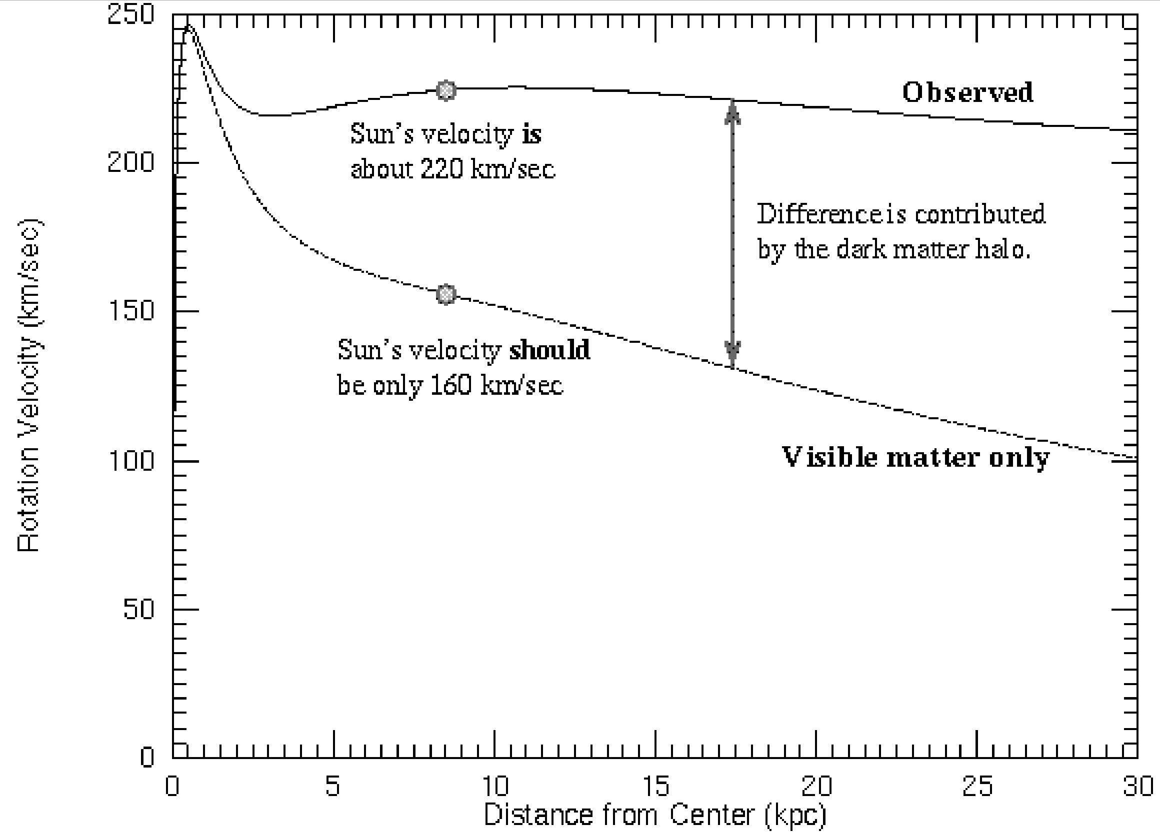

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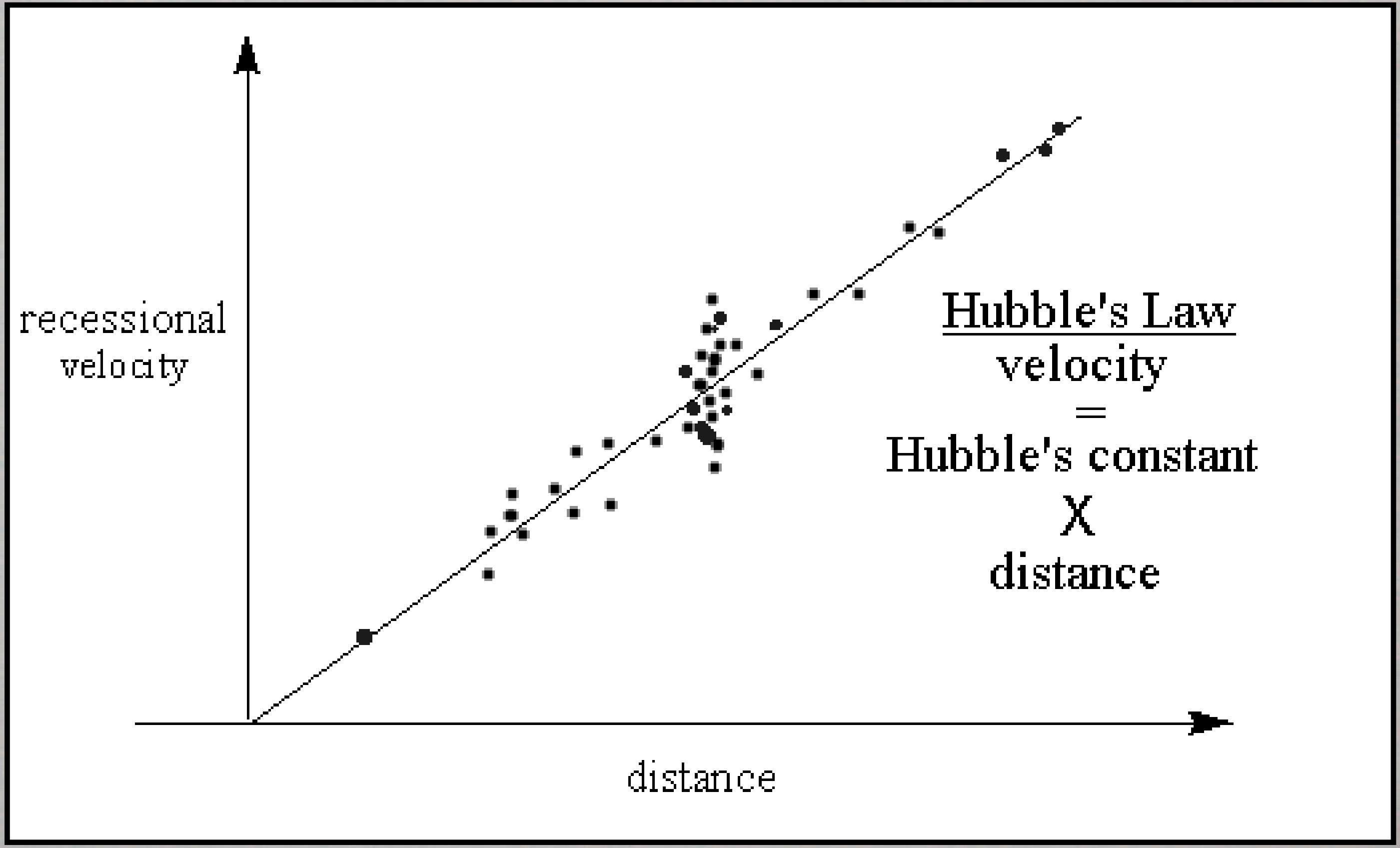
The Expansion of the Universe

$$v = H_0 d$$



The Expansion of the Universe

$$d = \frac{cz}{H_0}$$

When we measure the red shift we can calculate the distance.

$$H_0 = 70 \frac{km}{s} \Big/ Mpc$$

The Big Bang

$$t = \frac{x}{v} = \frac{d}{H_0 d} = \frac{1}{H_0}$$

Using the current estimate of H_0 we get an age of the universe of 14 billion years.

How to explain stars with math...

$\frac{dM}{dr} = 4\pi r^2 \rho(r)$	Mass Continuity
$\frac{dP}{dr} = \frac{GM(r)}{r^2}\rho(r)$	Hydrostatic Equilibrium
$U \propto e^{-E/kT - b/E^2}$	Energy Generation (rate/atom)
$P = \frac{\rho}{m} kT$	Equation of State
$L(r) = \left[\frac{-16\pi r^2 \sigma T^3(r)}{\mathrm{K}'(r)\rho(r)}\right]\frac{dT}{dr}$	Transport

Conclusion

- This has been the tip of the iceberg – astrophysics is complicated!
- But it is very, very cool.
- Thank you.